Module-1: Linear Algebra (Theory of Matrices)

Short questions for 2 marks

1. Find the product of eigen values for matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$

2. Find the eigen values of adjA where

$$A = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$$

3. The sum of eigen values of a 3x3 matrix is 6 and the product of the eigenvalues is also 6. If one of the eigenvalues is 1, find the other two eigenvalues.

4. If $A = \begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}$, then find the eigenvalues of $6A^{-1} + A^3 + 2I$

5. Find the characteristic root of $A'^2 - 3A' + 4I$ where $A = \begin{bmatrix} 2 & 0 \\ 5 & 3 \end{bmatrix}$

6. Corresponding to which eigenvalue $(2,3,-2,-3)'$ is an eigenvector of

$$\begin{bmatrix} 1 & -4 & -1 & -4 \\ 2 & 0 & 5 & -4 \\ -1 & 1 & -2 & 3 \\ -1 & 4 & -1 & 6 \end{bmatrix}$$

Questions for 4 ,6 ,8 marks

1. If $A = \begin{bmatrix} 123 & 231 & 312 \\ 231 & 312 & 123 \\ 312 & 123 & 231 \end{bmatrix}$, prove that

(i) one of the eigenvalues of A is 666 (ii) If A is non-singular, then one of the eigenvalues of A is negative.

2. Show that the following matrix is diagonalisable. Also find the diagonal form and a diagonalising matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

3. Find the eigenvalues and eigenvector (or bases for eigenspaces) of $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$

4. Find the eigenvalues and eigenvector of $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$

5. Find the characteristic equation of the matrix A and verify that it satisfies Cayley-Hamilton theorem. Hence find A^{-1} and A^4 ,where A is $\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$.

6. Find the characteristic equation of the matrix A and find the matrix represented by $A^6 - 6\,A^5 + 9\,A^4 + 4\,A^3 - 12\,A^2 + 2A - I$, where $\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$

7. Show that the following matrices are diagonalisable. Also find the diagonal form and a diagonalising matrix

(i) $\begin{bmatrix} -17 & 18 & -6 \\ -18 & 19 & -6 \\ -9 & 9 & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

8. Prove that characteristic root of $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ are of unit modulus.

9. Find the characteristic equation of following matrices and

obtain the inverse $\begin{bmatrix} 1 & 2 & 4 \\ -1 & 0 & 3 \\ 3 & 1 & -2 \end{bmatrix}$

10. Given $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$, find the eigenvalues of A. Also find eigenvalues of $4A^{-1}$ and eigenvector of $A^2 - 4I$.

11. By using Cayley-Hamilton theorem find A^{-1} and A^{-2} where $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

12. Find the eigenvalues and bases for eigenspaces of $\begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix}$

13. Find the eigenvalues and eigenvector of $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$

14. Find the eigenvalues and eigenvector of $\begin{bmatrix} -3 & -9 & -12 \\ 1 & 3 & 4 \\ 0 & 0 & 1 \end{bmatrix}$

15. Find the eigenvalues and bases for eigenspaces of $\begin{bmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{bmatrix}$

Module-2: Complex Integration

Short Questions for 2 marks

1. If f(a)=$\int_c \frac{4z^2+z+5}{z-a} dz$ where c is an ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ then evaluate f(i) .

2. Find the value of the integral $\int_0^{1+i}(x^2-iy)\,dz$ along the path y = x .

3. Evaluate $\int_0^{1+i}(x^2+iy)(dz)$ along the path y = 0 where x varies from 0 to 1.

4. Evaluate using Cauchy's Integral formula $\oint_C \frac{dz}{z^3(z+4)}$ where C is the circle $|z|=2$.

5. Evaluate $\int_C \frac{Z+6}{z^2-4}\,dz$, where C is the circle $|z|=1$.

6. Evaluate $\int_0^{1+i} z\,dz$ along $y=x$.

7. Find the residue at the pole z =-1 of $f(z)=\frac{1}{(z+1)(z-2)^2}$.

8. If $f(z)$ is analytic inside and on closed curve C of simply connected region R and if $z=2$ be any point within C, then find $\int_C \frac{f(z)}{z-2}\,dz$.

9. Evaluate $\int_c \frac{7z-1}{(z-3)(z+5)} dz$, where c is the circle $|z|$ =1.

10. Identify the type of singularity of the function $f(z)=\frac{sinhz}{z^7}$

<u>Questions for 4,6,8 marks</u>

1. Obtain Laurent 's series for $\frac{2}{(z-2)(z-3)}$ $in\ the\ region$: $2<|z|<3$.

2. Evaluate $\oint_C \frac{sin\pi z^2+cos\pi z^2}{(z-2)(z-3)} dz$ where C is the circle $|z|$ =4.

3. Evaluate using Cauchy's Residue Theorem, where C is a curve $|z-1|=3$ for $\int_C \frac{2z+1}{(z-1)^2(z-3)}dz$

4. Evaluate the given complex integral $\int_0^{3+i}(\bar{z})^2 dz$ along a parabola $x=3y^2$.

5. Evaluate:$\int_c \frac{z^2}{(z-1)^2(z-2)} dz$ $;c\ is\ |z|=2.5$

6. Expand: f(z)=$\frac{7z-2}{z(z+1)(z-2)}$$about\ z=-1, for\ 1<|z+1|<3$ as a Laurent's Series .

7. Evaluate $\int_c \frac{sin\,\pi\,z^2+cos\,\pi\,z^2}{(z-1)(z-2)} dz\ ;c\ \ is\ \ \ |z|=3$

8. Obtain Laurent's series expansions of f(x) = $\frac{z-1}{z^2-2z-3}$; $|z|>3$

9. Evaluate $\int_C \left(xy+y^2\right)dx + x^2dy$ where C is the closed curve of the region bounded by $y=x$ and $y=x^2$.

10. Find Laurent's series for $f(z) = \frac{4z+3}{z(z-3)(z+2)}$ valid for

(i) $2<|z|<3$ (ii) $|z|>3$

11. Evaluate $\int_C \frac{z^2+3}{z^2-1}dz$ where C is circle $|z-1| = 1$.

12. Find all possible Laurent's expansion $\frac{z}{(z-1)(z-2)}$ about z = **-2.**

13. Using Cauchy's Residue theorem evaluate $\int_c \frac{sinsin\ 3z}{z+\frac{\pi}{2}}\ dz$.

14. Evaluate $\int_C \frac{z^2-3z+2}{(z-3)(z-4)}dz$, $C: |z| = 3.5$.

15. Evaluate the following integral using Cauchy-Residue theorem.

$I = \int_C \frac{z^2+3z}{\left(z+\frac{1}{4}\right)^2(z-2)}dz$ where C is the circle $\left|z-\frac{1}{2}\right| = 1$.

Module-3: Linear Algebra: Vector Spaces

Vector Spaces

Q1 State and prove Cauchy-Schwarz inequality in R^2.

Q2 Verify Cauchy-Schwartz inequality for the vectors $u=(2,1,1)$ and $v=(2,0,1)$

Q3 For real values of a, b and θ, show that $(a\cos\theta + b\sin\theta)^2 \le a^2+b^2$ using Cauchy-Schwarz inequality.

Q4 Let $V = F(-\infty,\infty)$ be the set of all real valued functions defined on $(-\infty,\infty)$. For any f and g and for any scalar k, we define (i) $f=g$ if and only if $f(x)=g(x)$ for all x
(ii) $(f+g)(x) = f(x)+g(x)$
(iii) $(kf)(x) = kf(x)$.
Then is V a vector space?

Q5 Examine whether the set of 2×2 matrices defined as $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ with usual addition of matrices and scalar multiplication is a vector space.

Q6 Show that any plane passing through the origin is a sub-space of R^3.

Q7 Is $W = \{(a, 1, 1) / a \in R\}$ a subspace of R^3?

Q8 Determine whether the following vectors span the vector space of all polynomials of second order. $P_1 = 1 - x + 2x^2$; $P_2 = 5 - x + 4x^2$
$P_3 = -2 - 2x + 2x^2$

Q9 Define: (i) Basis of vector space
(ii) Dimension of a vector space
(iii) Orthogonal set

Q10 Verify that the vectors $v_1 = \left(-\frac{3}{5}, \frac{4}{5}, 0\right)$, $v_2 = \left(\frac{4}{5}, \frac{3}{5}, 0\right)$, $v_3 = (0, 0, 1)$ form an orthonormal basis in R^3 w.r.t. the Euclidean inner product. Express the vector $(3, -7, 4)$ as a linear combination of v_1, v_2, v_3.

Q11 Let R^3 have the Euclidean inner product. Use Gram-Schmidt process to transform the basis $\{u_1, u_2, u_3\}$ into an orthonormal basis where $u_1 = (1, 1, 1)$, $u_2 = (-1, 1, 0)$, $u_3 = (1, 2, 1)$.

Q12 Let R^3 have the Euclidean inner product. Use the Gram-Schmidt process to transform the basis $\{u_1, u_2, u_3\}$ into orthonormal basis where $u_1 = (1, 0, 0)$, $u_2 = (3, 7, -2)$, $u_3 = (0, 4, 1)$.

Q13 Find an orthonormal basis for the subspaces of R^3 by applying Gram-Schmidt process where, $S = \{(1, 2, 0), (0, 3, 1)\}$

Q14 Check whether $\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}\right), \left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, 0\right)$ are orthagonal with respect to the Euclidean inner product.

Q15 Determine whether $v_1 = (2, -1, 3)$, $v_2 = (4, 1, 2)$, $v_3 = (8, -1, 8)$ span a vector space in R^3.

16. If a ,b ,c are three positive real numbers , then using Cauchy-Schwartz inequality prove that $(a + b + c)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \geq 9$

17. If $\|u + v\| = 7$ and $\|u - v\| = 3$ find $u.v$.

Module-4: Probability Distribution and Sampling Theory

<u>Short Questions for 2 marks</u>

1. If X is normal variate with mean 10 & standard deviation 4, then what is $P(X \leq 12)$? (Given: Area from z = 0 to z = 0.5 is 0.1915)

2. The means of two independent samples of size 8 and 7 are 1134 and 1024 respectively. The standard deviation of these two samples is 35 and 40 respectively. What is the value of test statistic t in order to test the significance of difference between sample means?

3. If X is a normal variate with mean 10 and standard deviation 4. The value of standard normal variate Z is

4. X is a Poisson Variate with mean 1.8. Then P[X≥1] is

<u>Questions for 4,6,8 marks</u>

Q.1 In sampling a large number of parts manufactured by a machine the mean number of defectives in a sample of 20 is 2. Out of 100 such samples, how many would you expect to contain 3 defectives using Poisson distribution.

Q.2 The marks obtained by students in a college are normally distributed with mean 65 and variance 25. If 3 students are selected at random from this college what is the probability that at least one of them would have scored more than 75 marks?

Q.3 Monthly salary X in a big organization is normally distributed with mean Rs. 3000 and standard deviation of Rs. 250 What should be the minimum salary of a worker in this organization so that the probability that he belongs to top 5% workers?

Q.4 In an intelligence test administered to 1000 students, the average was 42 and standard deviation was 24. Find the number of students (i) exceeding the score 50 and (ii) between 30 and 54

Q.5 An insurance company found that only 0.01% of the population is involved in a certain type of accident each year. If its 1000 policy holders were randomly selected from the population, what is the probability that no more than two of its clients are involved in such accident next year?

Q.6 Define Poisson distribution. Also state its mean and moment generating function.

Q.7 i) Define Normal distribution.
ii) State recurrance relation for Poisson distribution.

Q.8 Can we have a Poisson distribution with mean 4 and variance 5? Justify your answer.

Q:9 If X is a Poisson variate and $P(X=0) = 6\,P(X=3)$, find $P(X=2)$.

Q.10 In a distribution exactly normal 7% of items are under 35 and 89% are under 63. What are the mean and standard deviation?

11. In an exam taken by 800 candidates, the average and standard deviation of marks obtained (normally distributed) are 40% and 10% respectively. What should be the minimum score if 350 candidates are to be declared as passed.

12. A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as Poisson variate with mean 1.5. Calculate the proportion of day on which i) neither car is used ii) some demand is refused.

13. A certain drug administered to 12 patients resulted in the following change in their blood pressure.

5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4

Can we conclude that the drug increases the blood pressure?

14. When the first proof of 392 pages of a book of 1200 pages were read, the distribution of printing mistakes were found to be as follows.

No of mistakes in page (X)	0	1	2	3	4
No. of pages (f)	275	72	30	7	5

Fit a Poisson distribution to the above data and test the goodness of fit.

15. In an examination mark obtained by students in Mathematics, Physics and Chemistry are normally distributed with means 51, 53 and 46 with standard deviation 15, 12, 16 respectively. Find the probability of securing total marks i) 180 or above, ii) 80 or below

16. In a competitive examination the top 15% of the students appeared will get grade A, while the bottom 20% will be declared fail. If the grades are normally distributed with mean % of marks 65 and S.D. 10, determine the lowest % of marks to receive grade A.

17. Based on the following data determine if there is a relation between literacy and smoking

	Smokers	Non-smokers
Literates	83	57
Illiterates	45	68

(Given that Critical value of chi-square 1 d. f and 5% L.O.S is 3.841

18. A certain drug administered to 12 patients resulted in the following change in their Blood Pressure

5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4

Can we conclude that drug increase the Blood Pressure?

19. If a random variable X follows Poisson distribution such that

P(X=2) =9 P(X=4) +90 P(X=6)
Find the mean and variance of X.

20. Assume that the probability of an individual coal miner being killed in a mine accident during a year is 1/2400. Use appropriate statistical distribution to calculate the probability that in a mine employing 200 miners there will be at least one fatal accident every year.

21. If X is a normal variate with mean 30 and standard deviation 6, find the value of $X=x_1$ such that

$$P(X \leq x_1) = 0.05 \quad .$$

22. The income distribution of workers in a certain factory was found to be normal with mean of Rs 500 and standard deviation Rs 50. There were 228 persons above Rs 600. How many persons were there in all?

Module-5: Linear Programming Problems

Short questions for 2 marks

1. The Standard form of following LPP is

Minimise Z= -2x_1+x_2

Subject to $4x_1+5x_2 \geq -4$

$-3x_1+5x_2 \leq 7$

$x_1, x_2 \geq 0$

2. Find the dual of the following LPP

Maximize $5x_1 + 2x_2 + x_3$

Subject to ;

$3x_1 + x_2 + 7x_3 \leq 3$,

$x_1 + 4x_2 + 6x_3 \leq 5$

$x_1, x_2, x_3 \geq 0$

Questions for 4,6,8 marks

1. Solve by the Simplex method

Maximize $z = 10x_1 + x_2 + x_3$

Subject to $x_1 + x_2 - 3x_3 \leq 10$ $4x_1 + x_2 + x_3 \leq 20$

$x_1, x_2, x_3 \geq 0$

2. Find the dual of the following LPP

Maximize $z = 2x_1 - x_2 + 3x_3$

Subject to $x_1 - 2x_2 + x_3 \geq 4$; $2x_1 + x_3 \leq 10$; $x_1 + x_2 + 3x_3 = 20$

$x_1, x_3 \geq 0$ x_2 unrestricted.

3. Solve using dual simplex method

Minimize $z = 2x_1 + 2x_2 + 4x_3$

Subject to $2x_1 + 3x_2 + 5x_3 \geq 2$,

$3x_1 + x_2 + 7x_3 \leq 3$,

$x_1 + 4x_2 + 6x_3 \leq 5$

$x_1, x_2, x_3 \geq 0$

4. Using Simplex method solve the following LPP

Maximize $z = 5x_1 + 3x_2$

Subject to $x_1 + x_2 \leq 2$

$5x_1 + 2x_2 \leq 10$

$3x_1 + 8x_2 \leq 12$; $x_1, x_2 \geq 0$

5. Write the dual of the following LPP

$$Maximise \quad Z = 3x_1 + x_2 - x_3$$

$$Subject\ to \ \ x_1 + x_2 + x_3 \geq 8$$

$$2x_1 - x_2 + 3x_3 = 4$$

$$-x_1 + x_3 \leq 6$$

$x_1, x_3 \geq 0, x_2$is unrestricted.

6. Using Simplex method
Maximize $z = 10x_1 + 6x_2 + 5x_3$
Subject to $2x_1 + 2x_2 + 6x_3 \leq 300$
$10x_1 + 4x_2 + 5x_3 \leq 600$
$x_1 + x_2 + x_3 \leq 100$
$x_1,\ x_2, x_3 \geq 0$

7. Using the Big M method solve the following LPP
Maximize $z = 5x_1 - 2x_2 + 3x_3$
Subject to $2x_1 + 2x_2 - \ x_3 \geq 2$
$3x_1 - \ 4x_2 \leq 3$
$x_2 + 3x_3 \leq 5$
$x_1,\ x_2, x_3 \geq 0$

8. Determine all basic feasible solutions of the equations
$2x_1 + 6x_2 + 2x_3 + x_4 = 3$,
$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$

9. Construct the dual of the following problem,
Minimize $z = 2x_1 - \ x_2 + 3x_3$
Subject to $x_1 - 2x_2 + 3x_3 \geq 4$
$2x_1 + x_3 \leq 10$
$x_1 + x_2 + 3x_3 = 20$
$x_1, \ \ x_3 \geq 0$, x_2 $unrestricted.$

10. Using Simplex method
Maximize $z = 5x_1 + 3x_2 + 7x_3$
Subject to $x_1 + x_2 + 2x_3 \leq 26$
$3x_1 + 2x_2 + x_3 \leq 26$
$x_1 + x_2 + x_3 \leq 18$
$x_1, x_2, x_3 \geq 0.$
11. Using the Big M method solve the following LPP
Maximize $z = 4x_1 + 5x_2 + 2x_3$
Subject to $2x_1 + x_2 + x_3 \leq 10$
$x_1 + 3x_2 + x_3 \leq 12$
$x_1 + x_2 + x_3 = 6$
$x_1, x_2, x_3 \geq 0$

12. Construct the dual of the following problem,
Minimize $z = 2x_1 + 9\,x_2 + 11x_3$
Subject to $x_1 - x_2 + x_3 \geq 3$
$-3x_1 + 2x_3 \leq 1$
$2x_1 + x_2 - 5x_3 = 1$
$x_1, \quad x_2\,, x_3 \geq 0\,,$

13. Determine all basic feasible solutions of the equations
$2x_1 + 3x_2 + x_3 + 4x_4 = 8\,,$
$x_1 - 2x_2 + 6x_3 - 7x_4 = -3$

Module-6: Nonlinear Programming Problems

<u>Short questions for 2 marks</u>

1. The value of Lagrange's multiplier for the following NLPP is

Optimise Z=$7{x_1}^2 + 5{x_2}^2$

Subject to $2x_1$+$5x_2$=7

$x_1, x_2 \geq 0$

2. The value of Lagrange's multiplier λ for the following NLPP is

Optimize $z = 6x_1^2 + 5x_2^2$

Subject to $x_1 + 5x_2 = 7$

$x_1, x_2 \geq 0$

<u>Questions for 4,6,8 marks</u>

1. Obtain the relative maximum or minimum of the function
$z = 2x_1 + x_3 + 3x_2x_3 - {x_1}^2 - 3{x_2}^2 - 3{x_3}^2 + 17$

2. Maximize
$z = 6{x_1}^2 + 5{x_2}^2$

Subject to

$$x_1 + 5x_2 = 3$$

$$x_1, x_2 \geq 0$$

3. Optimize
$z = 2{x_1}^2 + 3{x_2}^2 + {x_3}^2$

Subject to $x_1 + x_2 + 2x_3 = 13$,$2x_1 + x_2 + x_3 = 10$

$$x_1, x_2 \geq 0$$

4. Using Kuhn-Tucker conditions
Maximize
$z = 7{x_1}^2 + 5{x_2}^2 + 6x_1$

Subject to $x_1 + 2x_2 \leq 10$

$$x_1 - 3\,x_2 \leq 9$$

$$x_1, x_2 \geq 0$$

5. Find the relative maximum or minimum of the function

$$z = {x_1}^2 + {x_2}^2 + {x_3}^2 - 8\,x_1 - 10x_2 - 12x_3 + 100$$

6. Using Lagrange's multiplier
optimize $z = 4\,x_1 + 6x_2 - 2\,{x_1}^2 - 2\,x_1\,x_2 - 2{x_2}^2$
subject to $x_1 + 2x_2 = 2$

$$x_1,\ x_2 \geq 0$$

7. Using Lagrange's multipliers solve

$$Optimise \quad Z = 3{x_1}^2 + 2{x_2}^2 + 4x_1 + 2x_2$$

$$Subject\ to\ \ 3x_1 + 5x_2 = 11$$

$$x_1, x_2 \geq 0$$

8. Solve the following NLPP by using Kuhn-Tucker conditions:

Maximize $z = 10x_1 + 4x_2 - 2x_1^2 - x_2^2$

Subject to $2x_1 + x_2 \leq 5$

$$x_1, x_2 \geq 0$$

9. Solve following NLPP using Kuhn-Tucker method

Maximize $z = 2x_1^2 - 7x_2^2 - 16x_1 + 2x_2 + 12x_1x_2 + 7$

Subject to $2x_1 + 5x_2 \leq 105$

$$x_1, x_2 \geq 0$$

10. Solve the following NLPP using Kuhn-Tucker conditions

$$Maximise \quad Z = 16x_1 + 6x_2 - 2{x_1}^2 - {x_2}^2 - 17$$

$$Subject\ to \quad 2x_1 + x_2 \leq 8$$

$$x_1, x_2 \geq 0$$

11. Using the method of Lagrange's multiplier solve the following NLPP

Optimize $z = 2x_1 + 6x_2 - x_1^2 - x_2^2 + 14$

Subject to $x_1 + x_2 = 4$; $x_1, x_2 \geq 0$

12. Using Lagrange's multipliers solve the following NLPP

Optimise $z = 4x_1 + 8x_2 - x_1^2 - x_2^2$

Subject to $x_1 + x_2 = 2$

$x_1, x_2 \geq 0$

13. Find the relative maximum or minimum of the function

$$z = -4{x_1}^2 - 9{x_2}^2 - 9{x_3}^2 + 2\,x_1 + 9x_2x_3 + 6x_3$$